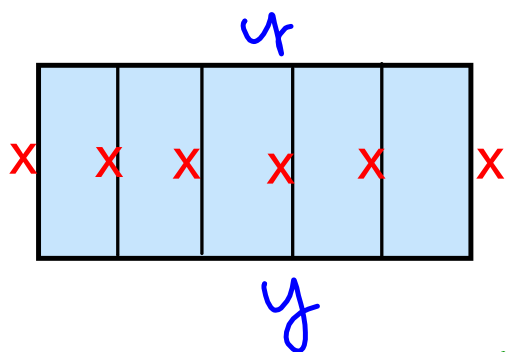
Precalc Warm Up # 8-4

1. The enclosure below must be built with 600 yards of fencing material. What dimensions would maximize the area? What is that area?



$P =$

$A = x(y)$

$6x + 2y = 600$

plug in $x = 50$

$y = 300 - 3x$

$y = 300 - 3(50)$

$\boxed{y = 150}$

$A = x(300 - 3x)$

2. Graph $y = -3x(x+2)^2(x-5)^3(x+1)$

$x = 0 \quad x = 100$

$A = 50(300 - 150)$

$A = 50(150)$

$(\boxed{50}, \boxed{\text{Area}})$

$(50, \boxed{7500})$

2. Graph $y = -3x(x+2)^2(x-5)^3(x+1)$

deg: 7 ⊖ ↑ ↓

$x \to -\infty,\ f(x) \to \infty$

$x \to \infty,\ f(x) \to -\infty$

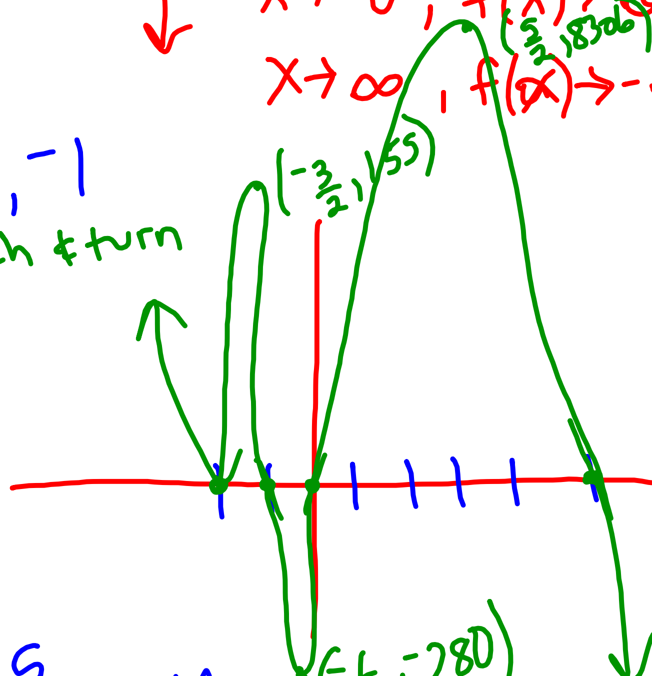
Zeros: $x = 0, -2, 5, -1$

touch & turn

$f\left(-\frac{3}{2}\right) = 155$

$f\left(-\frac{1}{2}\right) = -280$

$f\left(\frac{5}{2}\right) = 8306$



$x = -1.5$ $y =$

How would this graph be different?

$y = -3x(x+2)^2(x-5)^3(x+1)^4$

deg: 10 ⊖ ↓ ↓

Zeros 0, -2, 5, -1

touch & turn

HW Questions: p. 211

In Exercises 11–16, use the Rational Zero Test to list all possible rational zeros of f and verify that the zeros of f shown on the graph are contained in the list.

$\boxed{13.}$ $f(x) = -4x^3 + 15x^2 - 8x - 3$

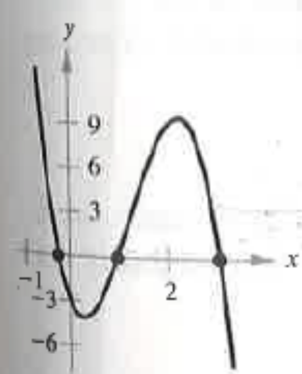


FIGURE FOR 13

In Exercises 21–36, find the real zeros of the given function.

$\boxed{21.}$ $f(x) = x^3 - 6x^2 + 11x - 6$

$\boxed{25.}$ $h(t) = t^3 + 12t^2 + 21t + 10$

$\boxed{27.}$ $f(x) = x^3 - 4x^2 + 5x - 2$

31. $f(x) = 4x^3 - 3x - 1$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$(x-1)(4x^2+4x+1)=0$

33. $f(y) = [4y^3 + 3y^2] + [8y + 6]$

$0 = y^2(4y+3) + 2(4y+3)$

$0 = (4y+3)(y^2+2)$

$\boxed{y = -\frac{3}{4}}$

$\sqrt{y^2} = \sqrt{-2}$

$$\begin{array}{|cccc} (4 & 3) & (8 & 6) \\ \hline \end{array}$$

35. $f(x) = x^4 - 3x^2 + 2$

$\pm 1 \quad \pm 2$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 2 \\ & \downarrow & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & 0 \end{array}$$

$(x-1)\ (x^3 + x^2)(-2x - 2)$

$(x-1)\ x^2(x+1) - 2(x+1)$

$(x-1)(x^2-2)(x+1) = 0$

In Exercises 37–44, find all real solutions of the given polynomial equation.

39. $x^4 - 13x^2 - 12x = 0$

$x(x^3 - 13x - 12)$

$\pm 1 \pm 2 \boxed{\pm 3 \pm 4} \pm 6 \pm 12$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -13 & -12 \\ & \downarrow & 4 & 16 & 12 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$x(x-4)(x^2+4x+3)$

$x(x-4)(x+3)(x+1)=0$

$\pm 1, \pm 2, \pm 3, \pm 4,$

$\pm$

43. $x^5 - 7x^4 + 10x^3 + 14x^2 - 24x = 0$

$x(x^4 - 7x^3 + 10x^2 + 14x - 24)$

$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

$x(x-3)(x^3 - 4x^2 - 2x + 8)$

$x(x-3)[x^2(x-4) - 2(x-4)]$

$x(x-3)(x-4)(x^2-2) = 0$

$x = 0, 3, 4, \pm\sqrt{2}$

In Exercises 45–48, (a) list the possible rational zeros of f, (b) sketch the graph of f so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of f.

a) $\dfrac{\pm 1 \quad \pm 2 \quad \pm 3 \quad \pm \cancel{6} \quad \pm \cancel{9} \quad \pm \cancel{18}}{\pm 1 \quad \pm 2 \quad \pm 4}$

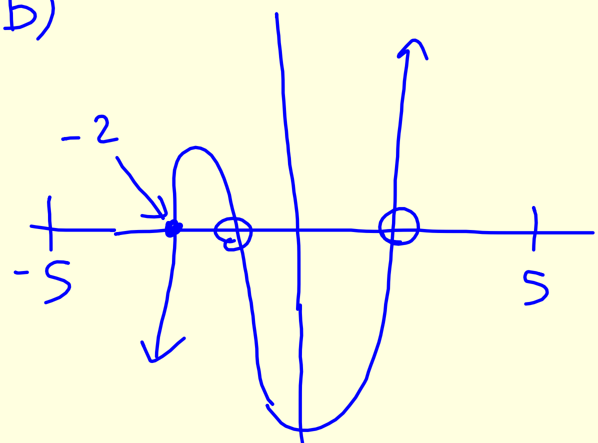
$0 = x^2(x - \frac{1}{4}) - 1(x - \frac{1}{4})$

$0 = (x - \frac{1}{4})(x^2 - 1)$

$\boxed{47.}$ $f(x) = 4x^3 + 7x^2 - 11x - 18$

$\boxed{51.}$ $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

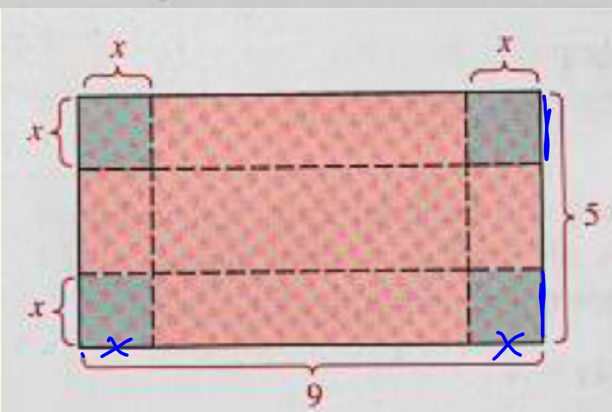
b)

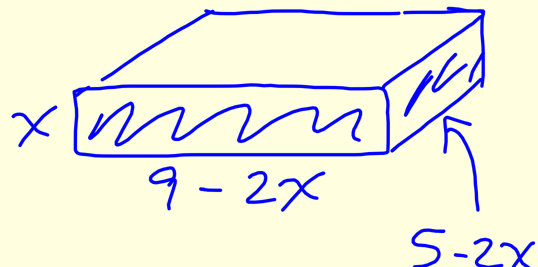


$(x+2)(4x^2 - x - 9) = 0$

$$\begin{array}{r|rrrr} -2 & 4 & 7 & -11 & -18 \\ & & -8 & 2 & \\ \hline & 4 & -1 & -9 & 0 \end{array}$$

53. An open box is to be made from a rectangular piece of material, 9 inches by 5 inches, by cutting equal squares from each corner and turning up the sides (see figure). Find the dimensions of the box, given that the volume is to be 18 cubic inches.





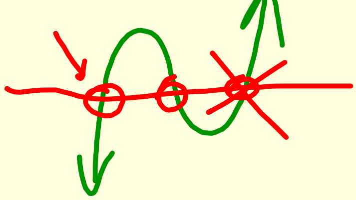
$$x(9-2x)(5-2x)=18$$

$$4x^3-28x^2+45x-18=0$$

$$\frac{\pm1 \pm2 \pm3 \pm6 \pm9 \pm18}{\pm1 \pm2 \pm4}$$

$x \approx .614$ $w \approx 3.77$

$l \approx 7.77$



Solve

$$[3x^3 + x^2] + [3x + 5] = 0$$

poss zeros $\frac{\pm 1 \quad \pm 5}{\pm 1 \quad \pm 3}$ $1, -1, 5, -5, \frac{1}{3}, \frac{-1}{3}$

$$\begin{array}{r|rrrr} -1 & 3 & 1 & 3 & 5 \\ & \downarrow & -3 & 2 & -5 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$(x+1)(3x^2-2x+5)$

use the discriminant

b^2-4ac

$\boxed{+}$

$4-4(3)(5)$

-56

$x = -1$ $\quad x = \frac{2 \pm \sqrt{-56}}{6}$

No real sol

Two of the solutions were COMPLEX.

real # imaginary #

$a + bi$ is called a COMPLEX NUMBER.

If $a = 0$ (and $b \neq 0$), it is said to be pure imaginary.

Recall that $i^2 = (\sqrt{-1})^2$	$i^5 = i^4 \cdot i^1 = 1 \cdot i = i$	$i^9 = i$
Therefore, $i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$	$i^{10} = -1$
$i^3 = i^2 \cdot i^1 = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot -i = -i$	$i^{11} = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = 1$

Find i^{842} $= \cancel{(i^4)^{210}} \cdot i^2 = -1$

$\cancel{4(210)} + 2$

Add, subtract, multiply, and divide complex numbers:

1. $(3+2i) + (-5-4i) = (3 + {}^{-}5) + (2i - 4i) = {}^{-}2 - 2i$

2. $(3 + 2i) + (+5 + 4i) = (3+5) + (2i+4i) = 8 + 6i$

3. $(3 + 2i)(-5 - 4i) = {}^{-}15 \boxed{-12i - 10i} - 8\boxed{i^2}$ $= {}^{-}1$

$\underbrace{{}^{-}15 - 22i + 8} = \boxed{{}^{-}7 - 22i}$

4. $\dfrac{(3+2i)}{(-5-4i)}$

On the last one, we need a "complex conjugate"

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4. $\dfrac{(3+2i)}{(-5-4i)} \cdot \dfrac{-5+4i}{-5+4i} = \dfrac{-15\boxed{+12i-10i}+8\boxed{i^2}^{=-1}}{(-5)^2-(4i)^2}$

$= \dfrac{-15+2i-8}{25-16\boxed{i^2}^{=-1}}$

$= \dfrac{-23+2i}{41}$

Find:

1. $\sqrt{25}$ $= 5$

2. $\sqrt{-16}$ $= \sqrt{-1} \cdot \sqrt{16} = \sqrt{i^2} \cdot 4 = 4i$

3. $\sqrt{-4} \cdot \sqrt{-25}$ $= \sqrt{i^2}\sqrt{4} \cdot \sqrt{i^2}\sqrt{25}$

4. $\sqrt{-3} \cdot \sqrt{-12}$

$2i \cdot 5i$

$10 i^2 = -1$

$\boxed{-10}$

5. $\sqrt{-48} - \sqrt{-27}$

6. $\left(-1+\sqrt{-3}\right)^2$

5. $\sqrt{i^2}\sqrt{16}\sqrt{3} - \sqrt{i^2}\sqrt{9}\sqrt{3}$

$4\boxed{i\sqrt{3}} - 3\boxed{i\sqrt{3}} = i\sqrt{3}$

HW: p 219 #2, 5-63 □

Quiz Monday:

PC 3.1 - 3.5